(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

MATHEMATICS—III

Time : 3 Hours *Total Marks : 100*

Note : Attempt **all** questions.

1. Attempt any **four** parts of the following : **(4×5=20)**

   (a) Define analytic function. Discuss the analyticity of $f(z) = \text{Re}(z^3)$ in the complex plane.

   (b) Show that $v(x, y) = e^{-x}(x \cos y + y \sin y)$ is harmonic. Find its harmonic conjugate.

   (c) Integrate $f(z) = \text{Re}(z)$ from $z = 0$ to $z = 1 + 2i$, (i) along straight line joining $z = 0$ to $z = 1 + 2i$, (ii) along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + 2i$.

   (d) Evaluate $\int_C \frac{(1+z)\sin z}{(2z-3)^2} dz$, where C is the circle $|z - i| = 2$ counter-clockwise.

   (e) Find all Taylor and Laurent series expansion of the following function about $z = 0$

   $$f(z) = \frac{-2z+3}{z^2-3z+2}.$$

   (f) Use contour integration to evaluate :

   $$\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}.$$

2. Attempt any **two** parts of the following : **(2×10=20)**

(a) Define moment generating function. Why is it called moment generating function ?

If $P(X=x)=\frac{1}{2^x}, x=1,2,3,\ldots\ldots\ldots.,$ find the moment generating function of x. Hence obtain the variance.

(b) Determine the normal equations if the curve $y = ax + bx^2$ is fitted to the data (x_i, y_i), i = 1, 2, ...m. Hence fit this curve to the data :

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

(c) Calculate the coefficient of correlation between the following ages of husband (x) and wife (y) by taking 30 and 28 as assumed mean in case of x and y respectively :

x	24	27	28	28	29	30	32	33	35	35	40
y	18	20	22	25	22	28	28	30	27	30	32

3. Attempt any **two** parts of the following : **(2×10=20)**

(a) Out of 800 families with four children each, how many families would be expected to have (i) 2 boys and 2 girls, (ii) at least one boy, (iii) no girl, (iv) at most two girls. Assume equal probabilities for boys and girls.

(b) Two groups of 100 people each were taken for testing the use of a vaccine. 15 persons contracted the disease out of the inoculated persons, while 25 contracted the disease in the other group. Test the efficacy of the vaccine using chi-square test. (The value of χ^2 for one degree of freedom at 5% level of significance is 3.84).

(c) Calculate the trend values by the method least square fit to a straight line and hence estimate profit for 1981 :

Year	1971	1972	1973	1974	1975	1976	1977
Profit (thousands)	60	72	75	65	80	85	95

4. Attempt any **four** parts of the following : **(4×5=20)**

(a) Explain Newton-Raphson method and use it to find the positive root of $x^4 = x + 10$ correct to three decimal places.

(b) Find the root of the equation $2x(1 - x^2 + x)\ ln\ x = x^2 - 1$, lying in the interval [0, 1] using Regula-Falsi method.

(c) Prove that $1+\delta^2\mu^2=\left(1+\frac{1}{2}\delta^2\right)^2$, where symbols have their usual meaning for finite differences.

(d) Use Newton-Gregory formula to interpolate the value of y at x = 36 from the following data :

x	21	25	29	33	37
y	18.4	17.8	17.1	16.3	15.5

(e) Find f(x) as a polynomial in x for the following data using Newton's divided difference formula :

x	−4	−1	0	2	5
f(x)	1245	33	5	9	1335

(f) Using Lagrange's interpolation formula, find polynomial which takes the values 3, 12, 15, −21 when x has the values 3, 2, 1, −1.

5. Attempt any **two** parts of the following : **(2×10=20)**

(a) Factorize the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

in the form LU, where L is lower triangular matrix and U is the unit upper triangular matrix and hence solve the system of equations :

$$4x_1 + x_2 + x_3 = 4;$$
$$x_1 + 4x_2 - 2x_3 = 4;$$
$$3x_1 + 2x_2 - 4x_3 = 6.$$

(b) (i) A slider in a machine moves along fixed straight rod. Its distances x(m) along the rod are given at various times (sec)

t	1.0	1.1	1.2	1.3	1.4	1.5
x	16.40	19.01	21.96	25.29	29.03	33.21

Find the velocity of the slider at t = 1.1 sec.

(ii) Evaluate $\int_0^{2x} (e^{-t} \sin 10t)\, dt$, using Simpson's rule with eight intervals.

(c) Given the initial value problem

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}, y(1) = 3.$$

Find the numerical solution for x = 1.2 and x = 1.5 by using Runge-Kutta method of fourth order.